

**LIGHT QUARK MASSES: A STATUS REPORT AT DPF 2000**

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A summary of the extraction of light quark masses from both QCD sumrules and lattice QCD simulations is presented. The focus is on providing a careful statement of the potential weaknesses in each calculation, and on integrating the work of different collaborations to provide a coherent picture.

**1. Introduction**

Significant progress has been made since 1996 in the determination of the light quark masses from both lattice QCD (LQCD) and QCD sumrules (SR). The evolution in time of the ranges quoted by the Particle Data Group<sup>1</sup> and our best estimates as of DPF2000 are given below. Since the scale dependence between 2 and 1 GeV is very large, we quote all results in the  $\overline{MS}$  scheme at scale  $\mu = 2$  GeV. Results at the lower scale 1 GeV may be obtained via  $m_q(1 \text{ GeV}) = 1.38 m_q(2 \text{ GeV})$  where the factor 1.38 corresponds to 4-loop running, with  $\alpha_s(m_\tau^2) = 0.334^2$ . In quoting separate values for  $m_u$  and  $m_d$  we have employed the ratio  $m_u/m_d = 0.553$ , obtained from ChPT. The basis for these estimates is outlined briefly below. A more detailed discussion will appear in an extended version of this writeup<sup>3</sup>.

	1996	2000	Proposed SR	Proposed LQCD
$m_u$	2 – 8 MeV	1 – 5 MeV	2.4 – 3.8 MeV	2.2 – 2.7 MeV
$m_d$	5 – 15 MeV	3 – 9 MeV	4.3 – 6.9 MeV	3.8 – 4.9 MeV
$m_s$	100 – 300 MeV	75 – 170 MeV	83 – 130 MeV	78 – 100 MeV

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## 2. QCD Sum Rule Determinations

We focus on recent ( $\geq 1995$ ) extractions of  $m_u + m_d$  and  $m_s$ , which employ either Borel (Laplace) transformed sum rules (BSR's) or finite energy sum rules (FESR's). For a typical correlator  $\Pi$  with spectral function  $\rho$ , BSR's approximate  $\rho(s)$  by its OPE version for  $s > s_0$ . Uncertainties in the the corresponding “continuum” spectral contribution are small only if  $s_0/M^2$  is significantly  $> 1$  for Borel masses,  $M$ , in the “stability window” of the analysis. In contrast, the spectral ansatz has an unphysical gap if  $s_0$  lies significantly beyond the squared mass of the last resonance appearing in the low- $s$  part of the ansatz. FESR's relate the integral from 0 to  $s_0$  of  $\rho(s)w(s)$  (with  $w(s)$  any analytic function) to the integral of  $\Pi(s)w(s)$  over the circle  $|s| = s_0$  in the complex  $s$ -plane. Using the OPE representation for  $\Pi$  in the latter integral is a potential problem since the OPE is expected to break down near the real timelike axis. A study of the isovector vector (IVV) channel using hadronic  $\tau$  decay data<sup>4</sup> shows that FESR's based on weights  $w(s) = s^k$  are *not* well-satisfied at scales  $\sim 2 - 3 \text{ GeV}^2$  but those based on “pinched weights” (those satisfying  $w(s_0) = 0$ ), which we will denote PFESR's, are, in contrast, very well-satisfied.

### 2.1. The Isovector and Isospinor Pseudoscalar Sum Rules

Since  $\partial^\mu A_\mu^{ij} = (m_i + m_j) : \bar{q}_i i \gamma_5 q_j :$ ,  $m_u + m_d$  and  $m_s + m_u$  can be determined from sum rules for the corresponding  $ij = ud$  and  $us$  correlators. Recent analyses (BPR<sup>5</sup> and its update, P98<sup>6</sup>, for  $ij = ud$  and DPS<sup>7</sup> for  $ij = us$ ) employ the 4-loop  $D = 0$  OPE expression. Known  $D = 4, 6$  contributions are at the few percent level, while other non-perturbative effects are assumed to be small. The  $\pi$  ( $K$ ) spectral contributions are known, but those from the (excited) resonance region are not. A modified sum of two Breit-Wigners is used to model the latter, the overall scale being set by normalizing the sum of resonance *tails* at threshold to values given by ChPT. P98 uses non-PFESR's ( $w(s) = 1$  and  $s$ ) and  $s_0 \sim 2 \rightarrow 3.5 \text{ GeV}^2$ ; DPS use the BSR framework and, for  $\Lambda_{QCD}^{(3)} = 380 \text{ MeV}$ ,  $s_0 \sim 6 \rightarrow 8 \text{ GeV}^2$ . The results are  $[m_u + m_d](2 \text{ GeV}) = 9.8 \pm 1.9 \text{ MeV}^6$  and  $m_s(2 \text{ GeV}) = 112 \pm 18 \text{ MeV}^7$ . PFESR's employing the BPR/P98 ansatz (tuned originally using *non-PFESR*'s) in the same range  $s_0 \sim 2 - 3 \text{ GeV}^2$  used by BPR/P98 yield very poor OPE/spectral integral matches<sup>4</sup>. Possible sources of this problem are the resonance normalization prescription and the use of non-PFESR's at scales where they are not well-satisfied in the IVV channel. Potential problems for DPS are the use of an input assumption about the relative strengths of the  $K(1460)$  and  $K(1830)$  contributions and the existence of a spectral gap between  $s \sim 4$  and  $6 \text{ GeV}^2$ . To shift focus to spectral normalization at the resonance peaks, rather than at threshold, an incoherent sum of Breit-Wigners can be employed, and a PFESR analysis used to fit both  $m_i + m_j$  and the resonance decay constants<sup>9</sup>. The resulting optimized OPE/spectral integral matches are at the  $\sim 1\%$  level or better in the fitting window,  $2.8 \rightarrow 3.6 \text{ GeV}^2$ , and correspond to  $[m_u + m_d](2 \text{ GeV}) = 9.9 \pm 1.0 \text{ MeV}$ ,  $m_s(2 \text{ GeV}) = 116 \pm 5 \text{ MeV}$ . The above results neglect direct instantons, and other non-perturbative effects not present in the OPE. Two examples show that non-trivial uncertainties are associated with this neglect. First, a phenomenologically-determined effective tachyonic gluon mass-squared, meant to represent additional short-distance non-perturbative contributions, lowers the P98 value by  $5.6\%$ <sup>8</sup>. Second, including instanton liquid model (ILM)<sup>10</sup> estimates of direct instanton effects, the PFESR extraction,  $[m_u + m_d](2 \text{ GeV}) = 9.9 \text{ MeV}$  is reduced to  $7.8 \text{ MeV}^9$ .

### 2.2. The Light-Strange Scalar Sum Rule

Since  $\partial^\mu V_\mu^{su} = i(m_s - m_u) : \bar{s}u :$ ,  $m_s - m_u \simeq m_s$  can be determined from sum rules

for the strange scalar channel. The low- $s$  part of  $\rho(s)$  in this case can be determined indirectly, *with certain additional theoretical assumptions*, from the Omnes representation of the timelike  $K\pi$  scalar formfactor,  $d_{K\pi}$ <sup>11</sup>. JM<sup>11</sup>, CDPS<sup>12</sup>, and CPS<sup>13</sup> employed this construction only to fix  $d_{K\pi}$  at threshold; the tail of a sum-of-Breit-Wigners was then normalized to this value, as in the BPR/P98 analyses. A purely resonant phase for  $d_{K\pi}$ , however, yields a slope,  $d'_{K\pi}(s=0)$ , incompatible with ChPT<sup>14</sup> and a poor OPE/spectral integral match<sup>4</sup>; both problems are removed if one includes background contributions to the phase near threshold and uses the Omnes representation for all  $s$ <sup>14,15</sup>. More recent analyses (J98<sup>16</sup>, M99<sup>15</sup>) employ the CFNP version of  $\rho(s)$ . CFNP and J98 are BSR analyses differing only in the value of  $s_0$ , while M99 is a PFESR analysis. CFNP has a good stability plateau for  $m_s$ , but at the cost of a gap in  $\rho$ . J98 has no spectral gap, but also no stability plateau, and also potentially non-trivial continuum contributions. The M99 PFESR's show an excellent OPE/spectral integral match. The results,  $m_s(2 \text{ GeV}) = 91 \rightarrow 116 \text{ MeV}$  for CFNP,  $116 \pm 22 \text{ MeV}$  for J98, and  $115 \pm 8 \text{ MeV}$  for M99, are all compatible. *Additional* errors associated with the assumptions made in writing down the Omnes representation are at least  $\sim 10 \text{ MeV}$  in magnitude.

### 2.3. Sum Rules Based on Electromagnetic Hadroproduction Data

Sum rules based on electromagnetic (EM) hadroproduction data<sup>17,18,19,20</sup> have significant uncertainties associated with either (1) isospin-breaking corrections<sup>18,19</sup> (for the flavor 33-88 vector current sum rule<sup>17</sup>) or (2) deviations from ideal mixing/Zweig rule violations<sup>21</sup> (for the flavor 33- $ss$  vector current sum rule<sup>20</sup>). We, therefore, consider these sumrules uncompetitive with those based on hadronic  $\tau$  decay and defer further discussion to the extended version of this write-up.

### 2.4. Sum Rules Based on Hadronic Tau Decay Data

The ratio of the hadronic  $\tau$  decay rate through the flavor  $f = ij = ud, us$  vector (V) or axial vector (A) current to the  $\tau \rightarrow \nu_\tau e \bar{\nu}_e$  rate,  $R_\tau^{V/A;ij}$ , can be written as a sum of appropriately-weighted integrals of the  $J = 0, 1$  parts of the corresponding hadronic spectral function. Since the  $ij = ud$  and  $us$  correlators are identical in the  $SU(3)_F$  limit, rescaled differences such as  $R_{\tau;ud}/|V_{ud}|^2 - R_{\tau;us}/|V_{us}|^2 \equiv \Delta R_\tau$  (with  $V_{ij}$  the  $ij$  CKM matrix element and  $R_{\tau;ij} \equiv R_\tau^{V;ij} + R_\tau^{A;ij}$ ) vanish in this limit. Experimental data allows access to the  $ud, us$  spectral distributions, and hence to integrals of such correlator differences, whose OPE representations, for large enough  $s_0$ , should be dominated by the  $D = 2$  term, proportional (neglecting  $m_{u,d}$ ) to  $m_s^2$ . The kinematic weights appearing on both sides of the  $\Delta R_\tau$  sum rule may be supplemented by a factor  $(1 - s/m_\tau^2)^k (s/m_\tau^2)^l$  without necessitating a  $J = 0/J = 1$  experimental decomposition; the analysis is then said to employ the “ $(k, l)$  spectral weight”.

The  $D = 2$  OPE representations of the  $J = 0 + 1$  and  $J = 0$  parts of the  $ud$ - $us$  correlator are known to 3-loop ( $O(\alpha_s^2)$ ) and 4-loop ( $O(\alpha_s^3)$ ) order, respectively. The  $ud$  spectral distributions are very accurately determined<sup>2</sup> while the  $V + A$  sum for the  $us$  case is known with errors of  $\sim 6 - 8\%$  in the  $K^*$  region and  $\sim 20 - 30\%$  above the  $K^*$ <sup>22</sup>. All analyses employ the ALEPH  $ud$  and  $us$  data. The CDH98<sup>23</sup>, MT98<sup>24</sup>, and CKP98<sup>25</sup> analyses are based on the preliminary result,  $R_{\tau,us} = 0.155 \pm .006$ ; ALEPH99<sup>22</sup>, PP99<sup>29</sup>, KKP00<sup>30</sup> and KM00<sup>21</sup> on the 1999 published version,  $0.1610 \pm .0066$ <sup>22</sup>; and DHPPC00<sup>26</sup> on a recent update,  $0.1630 \pm .0057$ <sup>26</sup>. Increasing  $R_{\tau,us}$  decreases  $R_{\tau;ud}$ , which is obtained via  $R_{\tau;ud} = R_{\tau;had} - R_{\tau;us} = [(1 - B_e - B_\mu)/B_e] - R_{\tau;us}$ . Small increases in  $B_e$  and  $B_\mu$ <sup>1</sup> have also lowered  $R_{\tau;had}$  since the earlier ALEPH analyses. The high degree of cancellation between  $ud$  and  $us$  spectral

integrals (typically to better than 10%) means the result for  $m_s$  is *very* sensitive to exactly which values of  $R_{\tau;ud}$ ,  $R_{\tau;us}$  have been used. For the same reason,  $m_s$  is also sensitive to small variations in the input values of  $f_K$  and  $|V_{us}|$  ( $|V_{us}|^2$ , e.g., differs by 2.6% depending on whether one uses the value from  $K_{e3}$  or that from 3-family CKM unitarity, combined with  $|V_{ud}|$  as extracted from  $0^+ \rightarrow 0^+$  nuclear decays; the difference is non-negligible on the scale of the  $< 10\%$   $ud-us$  residual). Different prescriptions for truncating the  $D = 2$  OPE series and estimating the truncation error also are present in the literature. In Table 1 we display the impact of converting existing results so as to correspond, where possible, to (1) the common truncation and error estimate scheme employed by DHPPC00 and (2)  $R_{\tau;us} = .163$ . A subtlety is involved in the second of these conversions for non-inclusive analyses and/or those employing  $s_0 \neq m_\tau^2$ ; see the extended version of this write-up for details. Since the  $O([\alpha_s^{eff}]^3)$  estimate in the KKP00 framework (which employs an effective running coupling and running masses) is not available to us, the KKP00 results have been converted only to reflect the new values of the  $ud$  and  $us$  integrals. Two converted versions are given in each case, one corresponding to the PDG2000 unitarity-constrained CKM set,  $|V_{ud}| = .9479$ ,  $|V_{us}| = .2225$  (CKMU), the other to the PDG2000 non-unitarity-constrained set,  $|V_{ud}| = .9735$ ,  $|V_{us}| = .2196$  (CKMN). Results from CDH98<sup>23</sup> and MT98<sup>24</sup> have been omitted since the former employed an older, erroneous expression for the integrated  $D = 2$ ,  $J = 0$  OPE contribution and the latter an assumption about the relation between the  $ud$  and  $us$   $J = 0$  spectral integrals now known to be invalid<sup>21</sup>. Also not quoted are the PP99 and ALEPH99 results, which have been superseded by the joint DHPPC00 update. The first error quoted is experimental, the second theoretical. The reader should bear in mind that the integrated  $D = 2$  OPE series for the  $J = 0$  contribution to the  $ud-us$   $\tau$  decay sum rules converges very badly<sup>24,27,28</sup> (for  $s_0 = m_\tau^2$  the ALEPH99 version, for example, is  $\sim 1 + 0.78 + 0.78 + 0.90 + \dots$ <sup>22</sup>). This creates potentially large theoretical errors for those (“inclusive”) analyses which retain both the  $J = 0+1$  and  $J = 0$  contributions (ALEPH99, CKP98, PP99, KKP00 and DHPPC00). While the identification, and hence subtraction, of the  $\pi$  and  $K$  pole  $J = 0$  contributions is unambiguous, at present no  $J = 0/J = 1$  separation exists for the experimental data in the excited resonance region. Analyses which subtract the  $J = 0$  contributions (ALEPH99, KM00), and work with the theoretically much better behaved  $J = 0+1$  contribution (the “0+1” approach), thus have instead  $J = 0$  subtraction uncertainties.

Our summary of estimates based on hadronic  $\tau$  decay is given in Table 1 along with the analysis type. Apart from KM00, these results neglect possible  $D > 6$  contributions. Because the polynomial coefficients grow rapidly with  $k$  for the  $(k, 0)$  spectral weights, this neglect becomes less safe as  $k$  increases. An extended discussion of the issues alluded to above, as well as details on the conversion of the individual results to the values quoted, will be given in the longer writeup<sup>3</sup>.

## 2.5. Summary

We consider results for  $m_s$  based on sum rules involving hadronic  $\tau$  decay data both most reliable and to have the greatest potential for improvement (through improved  $us$  spectral data which should be provided by the B factories). Results from other analyses are compatible, but with less controlled uncertainties. The agreement between all approaches, however, suggests that theoretical uncertainties are under reasonable control. Experimental errors in the different  $\tau$  analyses are necessarily strongly correlated, and hence cannot be averaged. Consistency of the different versions does, however, allow us to average the central values, and quote as our error the smallest of the errors shown (which corresponds to the least strong  $ud-us$  cancellation). With this prescription, we obtain for our final sum rule determination  $m_s(2 \text{ GeV}) = 114 \pm 16 \text{ MeV}$  for the CKMU input set and  $m_s(2 \text{ GeV}) = 101 \pm 18 \text{ MeV}$  for the CKMN input set. As mentioned above, theoretical uncertainties in  $m_u + m_d$

Table 1. Impact of conversion to common input for the hadronic  $\tau$  decay extractions of  $m_s$ . Results for  $m_s(2 \text{ GeV})$  are in MeV, and correspond to  $B_e = .1783$ ,  $B_\mu = .1737$ ,  $R_{\tau;us} = .163$ . All except KKP00 also correspond to the DHPPC00 truncation prescription, and to  $\alpha_s(m_\tau^2) = .334$ . Experimental and theoretical errors have been combined in quadrature.

Reference	Analysis Type	$m_s(2 \text{ GeV})$ (MeV)		
		Original	CKMU input	CKMN input
CKP98	inclusive, (0, 0)	$145 \pm 36$	$116 \pm 31$	$99 \pm 34$
KKP00	inclusive, (0, 0)	$125 \pm 28$	$120 \pm 28$	$106 \pm 32$
KM00	(0 + 1), $w_{20}$ <sup>21</sup>	$115 \pm 17$	$110 \pm 16$	$100 \pm 18$
DHPPC00	inclusive, (0, 0)	$126 \pm 31$	$124 \pm 32$	$106 \pm 37$
DHPPC00	inclusive, (1, 0)	$115 \pm 19$	$113 \pm 19$	$102 \pm 21$
DHPPC00	inclusive, (2, 0)	$100 \pm 21$	$99 \pm 21$	$91 \pm 21$

are larger. Using the ChPT ratio  $2m_s/[m_u + m_d] = 24.4 \pm 1.5$ , the CKMU and CKMN  $m_s$  results correspond to  $[m_u + m_d](2 \text{ GeV}) = 9.3 \pm 1.4$  and  $8.3 \pm 1.6$  MeV, respectively. For comparison, the P98 estimate is either 9.8 MeV or, if one uses the P98 1 GeV value together with our scaling factor,  $12.8/1.38 = 9.3$  MeV, while the PFESR/ILM estimate is 7.8 MeV. For our final summary we choose the conservative combined ranges 6.7–10.7 MeV for  $m_u + m_d$  and 83–130 MeV for  $m_s$ . These, as we show next, are in excellent accord with LQCD estimates.

### 3. Lattice QCD: Quenched Results

LQCD simulations provide estimates for two quantities  $\overline{m} \equiv (m_u + m_d)/2$ , and  $m_s$ .  $m_u$  and  $m_d$  are not obtained separately since current simulations do not include EM effects. Details of how quark masses are extracted from lattice calculations are given in <sup>31–39</sup> and will not be repeated here. We start with a summary of the state-of-the-art results given in Table 2 and discuss current shortcomings and future prospects. This table also provides information on (i) the lattice action used in the simulations, (ii) whether extrapolation to the continuum limit was done, and (iii) the states used to fix the quark masses and the lattice spacing  $a$ . The errors quoted include statistical and those due to renormalization constants and chiral and continuum extrapolations. Comparing data from different collaborations suggests that the quoted statistical errors are realistic and much larger than finite volume corrections. The latter three systematic errors will be discussed later.

At first sight, these estimates suggest a lack of consistency between different lattice calculations. We will attempt to argue that this is not so. The key reason is that it is only for QCD with three dynamical light flavors (the physical theory) that we expect all simulations to give the same results once the extrapolation to the continuum limit ( $a = 0$  to remove discretization errors) has been carried out. So, before analysing the relative merits of the different calculations one has to address the question of concern to all: how valid is the quenched approximation, especially since the resulting theory is not even hermitian? From a practical point of view quenching introduces two main limitations. First, the spectrum of the quenched theory will not coincide with the experimentally observed one. The second limitation, discussed in <sup>40,41</sup>, and known as the problem of quenched chiral logs, is more subtle. In a nutshell the problem is that the quenched  $\eta'$  persists as a Goldstone boson in the chiral limit and its propagator has a single and double pole, consequently the chiral expansion of pseudoscalar meson masses, decay constants, and quark condensates develop enhanced logarithms that are singular in the chiral limit <sup>40,41</sup>. The first limitation implies that there will be no consistent set of quark masses which reproduces the observed spectrum in the quenched approximation. The second is relevant when extrapolating data to the physical  $u$  and  $d$  quark

Table 2. Recent results for quark masses.  $O(a)$  SW stands for non-perturbative  $O(a)$  improved Sheikhholeslami-Wohlert (SW) fermion action, Iwasaki for an improved gauge action, and DWF for Domain Wall Fermions. In the last column we give the quantity used to set the lattice spacing  $a$  and  $(a \rightarrow 0)$  indicates that results were extrapolated to the continuum limit.  $r_0$  is defined in terms of the force at 0.5 fermi and  $a(r_0) \approx a(M_\rho)$ .

	Action	$\bar{m}$	$m_s(M_K)$	$m_s(M_\phi)$	scale $1/a$ (GeV)
Summary (1997) <sup>31</sup>		3.8(1)(3)	99(3)(8)	111(7)(20)	$M_\rho$ ( $a \rightarrow 0$ )
APE (1998) <sup>32</sup>	O(a) SW	4.5(4)	111(12)		$M_{K^*}$ $a^{-1} \approx 2.7$
APE (1999) <sup>33</sup>	O(a) SW	4.8(5)	111(9)		$M_{K^*}$ $a^{-1} \approx 2.7$
JLQCD (1999) <sup>35</sup>	Staggered	4.23(29)	106(7)	129(12)	$M_\rho$ ( $a \rightarrow 0$ )
CPPACS (1999) <sup>34</sup>	Wilson	4.55(18)	115(2)	143(6)	$M_\rho$ ( $a \rightarrow 0$ )
CP-PACS (2000) <sup>39</sup>	Iwasaki+SW	4.4(2)	110(4)	132(6)	$M_\rho$ ( $a \rightarrow 0$ )
ALPHA-UKQCD (1999) <sup>36</sup>	O(a) SW		97(4)		$f_K$ ( $a \rightarrow 0$ )
QCDSF (1999) <sup>38</sup>	O(a) SW	4.4(2)	105(4)		$r_0$ ( $a \rightarrow 0$ )
QCDSF (1999) <sup>38</sup>	Wilson	3.8(6)	87(15)		$r_0$ ( $a \rightarrow 0$ )
RIKEN-BNL (2000) <sup>37</sup>	DWF		105(6)(21)	127(6)(25)	$M_\rho$ $a^{-1} \approx 2.1$
CP-PACS (2000) <sup>39</sup>	$n_f = 2$ Iwasaki+SW	$3.44^{+0.14}_{-0.22}$	$88^{+4}_{-6}$	$90^{+5}_{-11}$	$M_\rho$ ( $a \rightarrow 0$ )
QCDSF-UKQCD (2000) <sup>45</sup>	$n_f = 2$ O(a) SW	3.5(2)	90(5)		$r_0$ [1.9 – 2.2]

masses, where the effects of the artifacts (enhanced chiral logs) becomes significant. So, in fact, the validity of quenched calculations will be judged *a posteriori* by comparison with  $(2 + 1)$  flavor calculations.

Quenched LQCD simulations proceed with the assumption that the stable particle masses and decay constants are affected at roughly the 10% level, *i.e.* the deviations are “small”. Within this assumption, the quenched results can be presented in one of two ways: ( $\mathcal{P}_1$ ) analyze the data in all possible ways and take the variation in the estimates as a measure of the quenching uncertainty, or ( $\mathcal{P}_2$ ) find combinations of quantities for which quenching effects are expected to be small, and use only these to extract a quenched number (which is then presumably “closest” to the real world). The results in Table 2 represent both approaches; for example, calculations by JLQCD and CP-PACS are closer to  $\mathcal{P}_1$ , while ALPHA-UKQCD follow the  $\mathcal{P}_2$  philosophy.

We start with three consensus statements regarding the quenched data in Table 2. First, for a given gauge and fermion action, the raw lattice data, and the masses of hadrons as a function of the bare quark masses, are consistent between the different calculations. Second, there is  $\sim 10\%$  uncertainty in the values of quark

masses depending on what physical quantity ( $M_\rho$ ,  $M_{K^*}$ ,  $f_K$ ,  $f_\pi$ , or  $r_0$ ) is used to set the lattice scale. (In this case, for example, proponents of  $\mathcal{P}_2$  would argue that  $M_\rho$  should not be used to set  $a$  because of its large width and similarly large shift in the mass between the unmixed and physical state.) Third, there is  $\sim 20\%$  uncertainty in  $m_s$  depending on whether  $M_K$  or  $M_{K^*}$  or  $M_\phi$  is used to fix it ( $M_{K^*}$  and  $M_\phi$  give consistent results). Furthermore, as expected, neither  $m_s(M_K)$  nor  $m_s(M_\phi)$  yield the observed splittings in the baryon decuplet<sup>42,34</sup>. Our view, in summary, is that these variations, with the states used to fix  $a$  and the quark masses, provide a very useful handle for evaluating the “reality” of coming simulations with dynamical quarks: these differences should vanish as the input dynamical quark masses are tuned closer and closer to their physical values.

Three other factors give rise to some of the differences observed in Table 2. These are (i) chiral extrapolations (ii) continuum extrapolations, (iii) the renormalization factor connecting lattice quark masses to those in the  $\overline{\text{MS}}$  scheme at  $\mu = 2$  GeV. A brief discussion of these is as follows.

*Chiral Extrapolations:* Simulations with physical masses for  $u$  and  $d$  quarks are computationally too expensive and beyond the capability of today’s computers. Consequently one simulates the theory over a range of quark masses, typically  $2m_s \rightarrow m_s/3$ , and extrapolates to  $\overline{m}$  using predictions of (quenched, or partially quenched for  $n_f = 2$ ) chiral perturbation theory.

*Continuum extrapolations:* The lattice theory at scale  $a$  has discretization errors of  $O(\alpha_s^i a^n)$ , where  $i, n$  depends on the order of improvement of the lattice action and operators. These errors are removed by extrapolating results calculated over a range of scales, typically  $2 \leq 1/a \leq 4$  GeV, to  $a = 0$ .

*Renormalization Constants:* These connect the lattice quark masses to either the renormalization group invariant mass<sup>36</sup>, or those in a continuum renormalization scheme like  $\overline{\text{MS}}$  at some fixed scale. These factors have been estimated using perturbation theory (CP-PACS), semi non-perturbatively (APE), and fully non-perturbatively (ALPHA-UKQCD); the latter being the most accurate.

One way to improve the reliability of the chiral and continuum extrapolations is to use quantities (or their ratios) that have the least ambiguity and dependence on  $m$  or  $a$  ( $\mathcal{P}_2$  approach). For example, while the statistical signal for extracting  $M_\rho$  is good, using it to set the lattice spacing can lead to a  $\sim 10\%$  uncertainty since the mass of the stable  $\rho$  calculated in lattice simulations could differ by  $\sim 100$  MeV from the physical value. It is therefore desirable to use ratios with the best statistical signal, and the smallest variation with the quark masses or  $a$  (see Ref.<sup>36</sup> for discussion).

It is not straightforward to assess the absolute magnitude of these three errors, independent of the analyses. The reason is that they are correlated and depend on the details of how each analysis was done. For example, consider doing a continuum extrapolation with simulations at  $1/a = 2, 3$ , and  $4$  GeV, scales typical of the best calculations. If the relative (systematic) error at the three points, due to say the chiral extrapolation, is not the same, then the bias can get magnified by the continuum extrapolation. From a survey of the various calculations, our estimate of the range of uncertainty in  $m_s$  from these three effects varies between  $2 - 15$  MeV, with the lower value coming from the ALPHA-UKQCD result which includes a fully non-perturbative calculation of the renormalization constant, and considers a kaon composed of two quarks of mass  $\approx m_s/2$  rather than  $m_s$  and  $m_u$ . The latter simplification avoids the need for chiral extrapolation in  $m_u$  and is justified on the basis that the chiral fit,  $M_\pi^2$  versus  $m$ , shows a linear behavior in the range  $2m_s - m_s/2$ .

Quenched calculations have significantly improved our understanding of, and ability to control, these three sources of systematic errors. For example, (i) use of improved actions has lead to a significant reduction in discretization errors (with even just  $O(a)$  improvement, the residual error now is only  $5 - 10\%$  at  $1/a \sim 2$  GeV);

(ii) development of reliable methods for calculating the renormalization constants non-perturbatively has reduced the associated uncertainty to  $< 2\%$  <sup>36</sup>; and (iii) partially quenched ChPT <sup>43</sup> has provided us with a much improved understanding of how to extract physical results from heavier quarks (in the range  $2m_s - m_s/4$ ), especially in the case of future  $2 + 1$  dynamical flavor simulations. To summarize, our bottom line, based on quenched simulations, is

- The technology for all aspects of the calculation has been developed.
- We have a very good understanding of statistical and systematic errors.
- The current best estimates are:  $90 \leq m_s \leq 140$  MeV and  $3.8 \leq \overline{m} \leq 4.8$  MeV.

#### 4. Lattice QCD: Results with 2 dynamical flavors

Until very recently, the least controlled systematic uncertainty, as discussed above, was that due to the quenched approximation, which was forced on us by limitations in computer resources. As of this year a number of collaborations (CP-PACS, JLQCD, APE, QCDSF-UKQCD, MILC, SESAM) have begun to report results from simulations with two dynamical flavors. These were reviewed at Lattice 2000 by Lubicz <sup>44</sup> and we discuss them briefly later. For the most part, we restrict our attention to the recent published results from the CP-PACS collaboration <sup>39</sup> (see Table 2) as these results are of quality similar to the quenched calculations in terms of statistics, lattice sizes, and range of  $a$  values explored.

The CP-PACS results are remarkable in that they show amazing consistency between  $m_s(M_K)$  and  $m_s(M_\phi)$ , in contrast to the  $\sim 20\%$  difference seen in the quenched theory. Also, compared to the quenched results of the ALPHA-UKQCD collaboration (who analyze data under the  $\mathcal{P}_2$  philosophy), these are  $\approx 10\%$  lower; the “expected” size of quenching errors.

There, however, remain issues that need to be better understood. First, comparing CP-PACS quenched and unquenched calculations with the same gauge and fermion action and with similar analysis methods, the unquenching effect is much  $> 10\%$ . For example,  $m_s(M_K)$  changes from  $110(4) \rightarrow 88^{+4}_{-6}$  MeV and  $m_s(M_\phi)$  from  $132(6) \rightarrow 90^{+5}_{-11}$  MeV! Second, extrapolations in sea quark mass from “heavy” dynamical  $u$  and  $d$  quarks, *i.e.* in the range  $2m_s \rightarrow m_s/2$ , yield already (within the statistical uncertainty) very good agreement between  $m_s(M_K)$  and  $m_s(M_\phi)$ . One needs to check, however, that this agreement persists when the partially quenched analyses recommended in <sup>43</sup> are fully implemented. Third, the role of the (so-far) neglected strange sea quarks remains to be investigated. Note that so far all lattice calculations indicate that the effect of adding dynamical flavors is to lower the estimates of quark masses. Fourth, the renormalization constants used for all  $n_f = 2$  estimates are perturbative; the corresponding non-perturbative calculations have yet to be done.

In spite of these cautionary remarks, we consider the CP-PACS numbers the current best lattice estimates. We quote these as  $[m_u + m_d](2 \text{ GeV}) = 6.8 \pm 0.8$  MeV and  $m_s(2 \text{ GeV}) = 89 \pm 11$  MeV, where we have doubled the error on  $m_u + m_d$  quoted by CP-PACS, based on that on  $m_s$ .

Finally, a brief comment on how CP-PACS numbers compare with preliminary data reported at LATTICE 2000 (see review by Lubicz <sup>44</sup>). The numbers vary from  $m_s = 90(5)$  MeV (QCDSF-UKQCD <sup>45</sup>) to  $m_s \approx 110$  MeV (APE <sup>44</sup>, MILC <sup>44</sup>). We expect that within the next couple of years a number of collaborations will have  $n_f = 2$  flavor results of numerical quality similar to those by CP-PACS. We are therefore confident that a consistent picture of light quark masses will emerge soon.



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